

A Note on Orthotropic Annular Circular Plate of Uniform Stress

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1. INTRODUCTION

Tang [2], [3], has obtained the stress-distribution in rotating orthotropic-annular disc for the constant thickness case. Murthy and Sherbourne [4] have determined the state of stress for the variable thickness case of the rotating orthotropic disc. It is of interest to look for the thickness function for such a disc which gives rise to uniform stress. This would be ideal for fully stressed-design of such plates. In this note such thickness function corresponding to the uniform velocity loading condition has been obtained.

NOTATION

- $\epsilon_r, \epsilon_\theta, \gamma_{r\theta}$ = Strains in polar coordinates with centre of the plate as the origin.
 $\sigma_r, \sigma_\theta, \tau_{r\theta}$ = Stresses in polar coordinates
 E_r, E_θ = Elastic Moduli in the radial and circumferential directions, respectively
 $\mu_{r\theta}, \mu_{\theta r}$ = Poisson's ratios
 u = Displacement in radial direction
 $\bar{E} = \frac{E_\theta}{E_r}$
 h = Thickness of the plate
 $\xi_1 = r_1/r_0$
 $\xi = r/r_0$
 ω = Angular velocity
 ρ = M/volume
 F_r = Body force per unit volume in r direction

2. GOVERNING EQUATIONS

For the two dimensional orthotropy in polar coordinates, the stress-strain relations are given below :

$$\begin{aligned} \epsilon_r &= \frac{\sigma_r}{E_r} - \mu_{r\theta} \frac{\sigma_\theta}{E_\theta} \\ \epsilon_\theta &= \frac{\sigma_\theta}{E_\theta} - \mu_{\theta r} \frac{\sigma_r}{E_r} \end{aligned} \quad (1)$$

where

$$\mu_{\theta r} E_\theta = \mu_{r\theta} E_r$$

Kinematic relations for axi-symmetric deformation are given below

$$\epsilon_r = \frac{du}{dr} ; \epsilon_\theta = \frac{u}{r} \quad (2)$$

Compatibility equation which readily follows from Eqn. (2) is

$$\epsilon_r - \frac{d}{dr} (r \epsilon_\theta) = 0 \quad (3)$$

The equilibrium equation in r , direction for the circular disk of variable thickness is as follows : [1]

$$\frac{d}{dr} (hr \sigma_r) - h \sigma_\theta + hr F_r = 0 \quad (4)$$

When the disk is rotating at uniform angular velocity, the body force F_r is given by

$$F_r = P \omega^2 r \quad (5)$$

The above set of equations give the complete formulation of the problem.

3. UNIFORM NORMAL STRESSES

In this problem, we have to stipulate that σ_r and σ_θ are constants for all values of ξ . Let $\sigma_r = \sigma_1$ and $\sigma_\theta = \sigma_2$ where σ_1 and σ_2 are constants. The compatibility equation (3) written in terms of stresses becomes

$$r \frac{d}{dr} [\mu_{r\theta} \sigma_r - \sigma_\theta] + (\mu_{r\theta} + \bar{E}) \sigma_r - (1 + \mu_{r\theta}) \sigma_\theta = 0$$

If in the above, $\sigma_r = \sigma_1$ and $\sigma_\theta = \sigma_2$, it follows that

$$\sigma_2 = \frac{\bar{E} + \mu_{r\theta}}{1 + \mu_{r\theta}} \sigma_1 \quad (6)$$

Thus, for a disk of uniform stress, the tangential normal stress σ_θ is $\frac{\bar{E} + \mu_{r\theta}}{1 + \mu_{r\theta}}$ times the value of the radial normal stress σ_r . Introducing equation (6) into the equilibrium equation (4) results in the following differential equation for the thickness function h :

$$\frac{dh}{d\xi} + \frac{1 - \bar{E}}{1 + \mu_{r\theta}} \frac{h}{\xi} + \frac{P \omega^2 r_0^2}{\sigma_1} h \xi = 0 \quad (7)$$

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whose solution is

$$\frac{h}{h_0} = \xi^p \exp \left[\frac{P\omega^2 r_0^2}{2\sigma_1} (1 - \xi^2) \right] \quad (8)$$

where, $p = \frac{\bar{E} - 1}{1 + \mu_{r\theta}}$ and h_0 is the thickness at the

station $\xi = 1$ (i.e. $r = r_0$). It is interesting to note that for the isotropic case ($\bar{E} = 1$) equation (6) reduces to $\sigma_2 = \sigma_1$ and equation (8) simplifies to

$$\frac{h}{h_0} = \exp \left[\frac{P\omega^2 r_0^2}{2\sigma_1} (1 - \xi^2) \right] \quad (9)$$

From (1) using equation (6) we obtain the following:

$$U = \frac{\sigma_1}{Er} \left(\frac{1 - \frac{\mu_{r\theta}^2}{\bar{E}}}{1 + \mu_{r\theta}} \right) r \quad (10)$$

CONCLUSIONS

Since it has been assumed that constant normal radial stress ($\sigma_r = \sigma_1$) exists at the boundaries as the boundary conditions, the thickness function obtained is valid only under above assumption.

Fig. 1 indicates the thickness variation for uniform normal stress (for typical K) for orthotropic as well as isotropic case. It is obvious from this figure that for $0.2 \leq \bar{E} \leq 1$, the thickness at the inner boundary is more than that at outer boundary. Also the variation in thickness is more for orthotropic case as compared to that of isotropic case. For $1 < \bar{E} \leq 10$ thickness is more at outer boundary as compared to one at inner boundary.

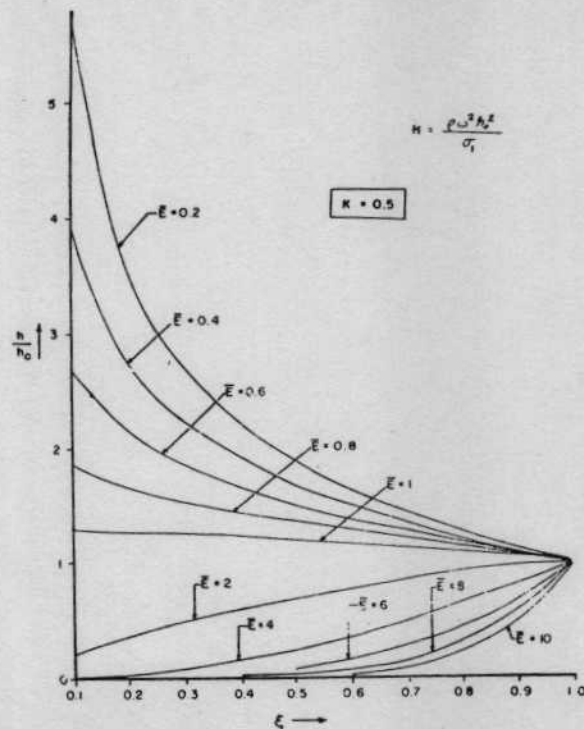


Fig. 1 Thickness Variation in Orthotropic Circular Annular Plate for Uniform Normal Stresses

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